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Boeing Aerospace Company
P.O. Box 3999
Seattle, Washington 98124

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Technical methods are defined for developing an integrated survival assessment for communication equipment in an environment consisting of the various effects generated by a nuclear burst. Consideration is given to analytical uncertainty and physical variability.

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SUMMARY

The objective of the Integrated Nuclear Communication Assessment (INCA) Program is to determine with known confidence the capability of the National Command Authorities (NCA) and military commanders to control forces in a nuclear conflict subject to nuclear weapon effects, and to determine those measures to assure that capability with known confidence.

Two quantities can be determined for the components within a facility during a nuclear effect assessment program. The first is the safety margin expressed as the dB ratio of the threshold of the component being assessed to the input to the component induced by the environment under consideration; the second is the data quality which is the statistical characterization of the uncertainties associated with the safety margin prediction values.

This document provides the results of studies which were performed to determine techniques for 1) defining the prediction uncertainties for nuclear effects, and 2) integrating assessment statements for various nuclear effects into one prediction relationship which describes the survival of a communication facility subjected to a combined nuclear environment.

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1.0 INTRODUCTION

The Defense Nuclear Agency (DNA) has undertaken an Integrated Nuclear Communications Assessment (INCA) Program which is a multiyear program involving several contractors and governmental agencies. The objective of the INCA Program is to determine with known confidence the capability of the National Command Authorities (NCA) and military commanders to control forces in a nuclear conflict, subject to nuclear weapon effects, and to determine those measures to assure that capability with known confidence.

The objective of the INCA Program is being met by: 1) developing the necessary analytical methodologies and data base, including test data, to permit an estimate to be made of the performance of Command and Control Information Systems (CCIS) in a nuclear warfare situation; and 2) applying the analytical methodology to evaluate the performance of selected CCIS elements in a nuclear scenario. The assessment methodology, analytical techniques, and data base are being structured to support the assessment of the worldwide CCIS capabilities in any nuclear scenario considering all nuclear weapon effects including blast, thermal, radiation, electromagnetic pulse (EMP), ground shock, dust, ejecta and magnetohydrodynamics (MHD).

1.1 BACKGROUND

Two quantities can be determined for the components within a facility during a nuclear effect assessment program. The first is the safety margin which is the dB-ratio of the threshold of the component being assessed to the input to the component induced by the environment under consideration; the second is the data quality which is the statistical characterization of the uncertainties associated with the safety margin prediction values. A technique has been developed (DNA 3859Z, "Survivability/Vulnerability Safety Margin Assessment" dated 30 September 1975) to determine statistical survivability using these two quantities.

The technique was applied following the analysis and testing of the Automatic Electric Company (AeCo) AUTOVON switching center located at Delta, Utah and the ESS-1 AUTOVON switching center located at Pickens, Mississippi. Electromagnetic pulse responses, expected to be induced during the test programs, were predicted using computer analysis techniques prior to the start of test activities. Following the tests, the test data and predicted response data were analyzed to define the data quality applicable to the electromagnetic models.

The technique used to determine the electromagnetic pulse (EMP) prediction data quality required the use of data obtained during test programs using EMP simulation sources. The INCA Program is scoped to include assessments for nuclear effects which cannot be adequately simulated during test programs. Thus, techniques must be developed such that a prediction data quality can be determined without the use of test data. Following the development of such techniques, the previously defined methods can be utilized to define a statistical assessment statement for a specific nuclear effect.

The INCA Program will be the first program which attempts to integrate several assessment statements, for various nuclear effects, into one overall assessment statement for a communication facility. Consequently, a technique must be developed to facilitate these assessment statement integrations.

1.2 SCOPE

This document provides the results of studies which were performed to determine techniques for 1) defining prediction data qualities for nuclear effects other than EMP, and 2) integrating assessment statements for various nuclear effects into one prediction relationship which describes the survival of a communication facility to a combined nuclear environment.

The techniques defined assume the availability of accurate input data and/or controlling equations. As for all analyses of physical phenomenon, the results developed using the techniques presented in this report will only be as accurate as the input data and/or equations used to model the interactions and mechanisms that control system survivability. This constraint does not limit the applicability of the techniques nor is the constraint unique to the techniques. For all mathematical analyses of physical phenomenon, the analyst must develop analytical equations and data which model the physical reality. Without knowing the actual model, the analyst cannot place "confidence" characterizations on the analytical model. Instead, he must assume the analytical model is correct and through use of the model illustrate that analysis results do not disagree with physical realities or known conditions. However, when the analysis results disagree with physical realities, the analysis techniques are not considered inadequate - the assumed model is considered incorrect and is modified.

The material contained in this document is introductory. That is, as with all introductory statistical material, liberal use of normal distribution and statistical independence assumptions are made to simplify presentations. The techniques presented can be used when data is not normally distributed or when variables are not statistically independent. However, evolution of the techniques requires advanced statistical methods and should be accomplished only on a case-by-case basis as needed.

2.0 DATA QUALITY/CONFIDENCE FOR EACH EFFECT

The material presented in this section defines techniques which can be used to characterize the data quality associated with a safety margin prediction method. The techniques defined vary from those which incorporate test data to those which use purely analytical methods.

2.1 TEST VERIFICATION

Determining prediction data quality by test verification involves predicting response, performing a test to measure the actual response, and using the differences between predicted and measured response to define prediction uncertainty. It is the most straightforward and intuitively satisfying method, but is expensive due to the required test activities. It should be pointed out that the value(s) obtained during a test are not the true value(s) of the quantities being measured; thus, the test measurement uncertainties must be included, if possible, during any analysis resulting in a definition of the predictions uncertainty based upon comparisons of predicted and test results.

2.1.1 Predictions

To determine the prediction uncertainties associated with a prediction method or technology, it is important to establish and define the procedures and level of effort which are included in the prediction method. This definition should include the qualifications of personnel developing the predictions, the time allotted to obtain the predictions, the theoretical and computational concepts used, the level of detail modeled, and the degree to which physical parameters are known or must be approximated. This definition should be established to assure that all concerned understand the technique to which the eventually determined prediction data quality may be applied.

In practice, predictions will be made for many points within a facility under examination. A subset of these points will be tested for the purpose of determining prediction uncertainties. Sample theory concepts should be applied to determine the sample size and the selection of the subset.

As a first step, the sample unit must be defined. This is usually a simple decision and consists of determining whether a sample unit consists of a component, a circuit, an equipment rack, or all racks of a given level of predictive complexity. Combinations of units may be selected. However, data quality characterizations should, whenever possible, be developed for homogeneous sample unit groups since the data quality varies group-to-group. As an example, the data quality for rack-level predictions is generally better than for component-level predictions since additional details must be characterized when moving the prediction points from the rack level to the component level.

Since testing to determine data quality is essentially a problem of estimating variance, the required sample size can be approximated for a nearly normal distribution by using the Chi-squared distribution if the required or desired accuracy is fixed. This statistical problem is treated in introductory mathematical statistics texts and is briefly addressed in Appendix A. For populations which are not normally distributed, or near-normally distributed, more advanced techniques are necessary to determine the required sample size. Such techniques are outside the scope of this report but can be found in advanced statistics texts.

The selection of the points for which predictions are made should assure, to the extent possible, that a representative sample is selected; techniques for selecting representative samples are defined in all statistics texts which address sampling theory. If more than one type of sample unit exists, the selected sample units should include the spectrum of types of sample units for which predictions are established. The proportions of types of sample units in the

sample should approximate the proportion of prediction unit types in the facility or weighted statistics should be utilized. This selection technique avoids intentional or inadvertent testing of a high proportion of units for which prediction variability is relatively low or high.

The selection of test measurement points should assure, to the extent possible, that a random sample is selected. The intent is to avoid a test sample that is different in an important way from the whole set for which the test results are expected to apply. Randomness can be assured in part if no special prediction analysis or review is conducted on test points that is not normally conducted for prediction efforts which are not verified by test.

Predicted responses may be given in terms of several variables or a continuum of values. An attempt should be made to reduce the response prediction to one "most-important" variable (failure power, voltage, current, predominant frequency, etc.), or a small number of variables, to allow a clear characterization of the important prediction uncertainies. Many techniques exist for determining the "most-important" variable; the technique commonly employed consists of utilizing partial differentiation procedures to determine which independent variable(s) significantly affect the dependent variable.

2.1.2 Tests

If more than one test method is available, the selection of the test method should take cost and the verified ability to meet accuracy requirements into consideration.

An analysis must be made to determine the uncertainties of the test measurement method. Bias is the difference between the average measurement and the true value of the quantity measured; a zero bias measurement method is desired such that the expected value of the quantity being measured is equal to the measurement value. Reproducibility of a measurement is defined by the variation

about the average for the same test point and test methods. The bias and reproducibility of the test measurement method should be used in the data quality analysis, and may be used in determining the desired number of measurement repetitions required if measurement averages are to be used to reduce test errors. Test data cannot be used to quantify prediction uncertainty unless the uncertainty bound of test measurements is significantly less than the expected prediction uncertainty.

2.1.3 Statistical Data Reduction

It will be assumed herein that the response prediction can be reduced to a "most-important" variable; that is, the variable which predominently influences the facility survivability. If the response predictions are affected by many variables, then the procedures identified must be expanded or modified to handle the more complex problem.

Usually only one prediction of the expected variable value will be made for each point, and this procedure will be assumed here, although expansion of the techniques presented herein to methods employing several predictions at a point is relatively simple. Let p_j be the prediction value for point j.

One or more test measurements will be made at each point; the general n-measurement case will be assumed herein. Let t_{ij}, i=1,2...,n represent test measurement i at point j.

For each test point, the mean test measurement value must be calculated. If later analysis includes a logarithm conversion, such as to the dB domain, the geometric mean $\left(\prod_{i=1}^{n} t_{ij}\right)^{1/n}$ should be calculated because the logarithm of the geometric mean is equal to the arithmetic mean of the test measurement value logarithms.

During test data reduction for a test point with n>2, a check should be made for outliers that may be due to bad test setups, mislabeling, or similar mistakes. If the t_{ij} values at a point are approximately normally distributed and σ is the repeatability standard deviation for the test method, t_* would be a suspected outlier (α= .001) if $|t_* - \bar{t}|>3.3\sigma$. If an outlier is detected, then the outlier should be examined to determine the reason(s) that the value does not conform with the remaining data. If there is good reason to believe that the outlier is an incorrect value, the mean can be approximated by using the median (reference 1). If several outliers are suspected in a large sample, these can be detected using special tests, and parameters can be estimated using trimmed or Winsorized estimators (reference 2).

The differences between predicted and measured responses can be defined by several different statistics. Examples are the statistics $p_j-\bar{t}_j$, $p_j/\bar{t}_j$, and log $(p_j/\bar{t}_j)$. The selection of which statistic to use depends on the possible values of p_j and $\bar{t}_j$, and the resulting distributional form of the chosen statistic. Analysis is usually simplified if the statistic chosen is approximately normally distributed.

For electronic communication systems, prediction errors tend to arise from many small multiplicative errors. This causes the logarithm of prediction errors to follow a normal distribution (reference 3).

In the following discussions, the logarithmic transformation to a decibel measure of error, $x_j = 20 \log (p_j/\bar{t}_j)$, will be used where j denotes the test points, j=1,2, ..., m, and t_j the geometric mean $\left(\prod_{i=1}^{n} t_{ij}\right)^{1/n}$.

As a first analysis step, the hypothesis that X is a normally distributed random variable must be statistically tested. If the hypothesis is rejected, investigation of alternative statistics or distributions must be performed. The test for normality should be a goodness-of-fit test such as the Cramér-vonMises test (reference 4), the W test (reference 3), or a Chi-squared test if the sample is quite large (reference 5).

Following the distributional tests, the hypothesis that $\mu_x=0$ (i.e., that there is no bias in the prediction method) must be examined. If the hypothesis is rejected, the prediction data quality characterization must be defined to include a prediction bias term.

The statistical test of the hypothesis that $\mu_x=0$ when x_j is a sample from a normal distribution is a standard textbook use of the student's t distribution (reference 3). If the hypothesis is rejected, the bias estimate is $\hat{b} = \frac{\Sigma x_j}{m}$ and the data quality statistic, defined as $DQ = 3\hat{\sigma}$, is characterized by $DQ = 3(\Sigma(x_j - \hat{b})^2/(m-1))^{1/2}$. If the hypothesis is accepted, the prediction data quality statistic can be defined as $DQ = 3(\Sigma\, x_j^2/m)^{1/2}$.

These data quality statistics include the errors due to the measurement system uncertainties as well as the uncertainty errors in the predictions. If the predictions fall within the range of the measurements, much of the prediction uncertainty defined may be due to measurement system acquisition errors. In order to obtain an estimate of prediction uncertainty with measurement errors removed, the following method of Taylor expansion about system function (reference 3) may be used:

Let:

$$x_j = h\,(p_j,\ t_{1j},\ t_{2j},\ \ldots,\ t_{nj}) = 20 \log\,(p_j/\bar{t}_j)$$

$$= 20 \log(p_j) - (20/n) \sum_{i=1}^{n} \log t_{ij}$$

and note that x_j is in the dB while p_j and t_{ij} are in the original units.

Using the expansion formula for $r = h\,(S_1, S_2, \ldots, S_K)$,

$$V_{ar}\,(r) = \sum_{i=1}^{n} \left(\frac{\partial h}{\partial s_i}\right)^2 \mathrm{Var}\,(S_i) + \sum_{i=1}^{n} \frac{\partial h}{\partial s_i}\,\frac{\partial^2 h}{\partial s_i}\,\mu_3(s_i)$$

results in a variance of x_j defined as:

$$\mathrm{Var}(x_j) = (20 \log e)^2 \left\{ (1/p_j)^2\,\mathrm{Var}\,(p_j) + (1/n^2) \left[\sum_{i=1}^{n} (1/t_i)^2 \right] \mathrm{Var}(t) \right\} + g(\mu_3)$$

where Var (t) is the uncertainty variance for the test measurements and $e = 2.71828\ldots$. The $g(\mu_3)$ terms may be dropped when p_j and t_{ij} are approximately symmetrically distributed random variables.

Then,

$$\mathrm{Var}\,(P_j) \cong p_j^{\,2} \left\{ \mathrm{Var}\,(x_j)/(20 \log e)^2 - \mathrm{Var}\,(t) \sum_{i=1}^{n} (1/t_i)^2/n^2 \right\}$$

for an individual prediction p_j at point j which has n test measurements.

Using $S_X{}^2$ for an estimate of Var (x_j) and pooling the estimates of Var (P_j) over the m prediction points gives:

$$\mathrm{Var}\,(p) = (1/m) \sum_{j=1}^{m} \mathrm{Var}\,(p_j)$$

$$= (1/m) \sum_{j=1}^{m} p_j^2 \left\{ S_x^2/(20 \log e)^2 - \mathrm{Var}(t) \sum_{i=1}^{n} (1/t_i)^2/n^2 \right\}$$

as an estimate of the prediction uncertainty variance with measurement errors removed.

If the test method uncertainty is much less than the prediction uncertainty, then

$$\mathrm{Var}(p) \cong (S_x^{\,2}\ \overline{p^2}/[20 \log e]^2)$$

If the test method uncertainty variability is much greater than the prediction uncertainty, then the estimate of Var (p) becomes unreliable and Var (p) may be estimated better by a subjective evaluation of its approximate value.

2.1.4 Data Quality

The above development leads to a statement of prediction uncertainty for a single response variable, such as the peak voltage response at a point due to electromagnetic simulation of a communications facility. Typically another variable must be predicted, such as the damage threshold of the component at the prediction point, in order to characterize the degree of safety which exists for the threat being analyzed.

For r and q in dB, if 1) Var (r) is the prediction uncertainty variance for the response, after adjusting for known bias, 2) Var (q) is the prediction uncertainty variance for the threshold, and 3) the degree of safety is characterized by a safety margin estimate, a, defined as a = q - r = 20 log (threshold/response), then the prediction uncertainty for the safety margin estimate is given by the variance Var (a) = Var (q) + Var (r).

If prediction errors for q and r are normally distributed, then the prediction errors for the safety margin estimate are normally distributed with standard deviation $\sigma_a = (\mathrm{Var}\ (a))^{1/2}$. As such, the probability that the true safety margin is greater than 0 (i.e., that the component at the prediction point is safe) is Pr(Y<a) for Y distributed as $N(0, \sigma_a)$.

2.2 METHOD OF MOMENTS

In some cases and for some effects, test validation of predictions is not feasible. However, if the equations used to make the predictions can be expressed in closed form and satisfy certain regularity conditions, and if the distributions of estimation errors of the independent variables are known, then the moments of the uncertainty probability distribution for the prediction value can be calculated.

The method used is the "method of generation of system moments." It consists of making a Taylor series expansion about the expected values of the independent variables. Once the distribution moments about the prediction value are estimated, the percentiles of the distribution can be estimated.

In theory, any number of higher moments can be estimated. In practical cases with limited data, the estimators of the higher moments of the probability should be limited to the first and second moments of the distribution of the system prediction values.

A more exact method, transformation of variables (reference 3, page 175), can be used on very simple problems. The method is not treated here because it can rapidly become very involved for moderately complex system functions.

2.2.1 Technique

Let the system prediction Z and the equation variables, $x_1, x_2, \ldots, x_n$, be related by the function $z=h(x_1, x_2, \ldots, x_n)$. Let $E(x_i)$ be the expected value or mean for the equation variable i, and let $\mu_K(x_i)$ denote its Kth moment about the mean ($\mu_K = E(x_i - \mu_1(x_i))^K$). Let $Var(x_i)$ be the variance or second moment.

If it cannot be assumed that all pairs of equation variables (x_i, x_j) are statistically independent, the Taylor series expansion gives

$$E(z) \cong h\,[E(x_1),E(x_2),\ldots,E(x_n)] + 1/2 \sum_{i=1}^{n} \frac{\partial^2 h}{\partial x_i^2}\,\mathrm{Var}\,(x_i)$$

$$+\sum_{\substack{i\;j\\ i<j}} \frac{\partial^2 h}{\partial x_i \partial x_j}\; E\left\{[x_i - E(x_i)][x_j - E(x_j)]\right\}, \text{ and}$$

$$\mathrm{Var}(z) \cong \sum_{i=1}^{n} \left(\frac{\partial h}{\partial x_i}\right)^2 \mathrm{Var}(x_i) + 2 \sum_{\substack{i\;j\\ i<j}} \left(\frac{\partial h}{\partial x_i}\right)\left(\frac{\partial h}{\partial x_j}\right) E\left\{[x_i - E(x_i)][x_j - E(x_j)]\right\}$$

$$+\sum_{i=1}^{n} \left(\frac{\partial h}{\partial x_i}\right)\left(\frac{\partial^2 h}{\partial x_j^2}\right) \mu_3(x_i)$$

$$+\sum_{\substack{i\;j\\ i\neq j}} \left(\frac{\partial h}{\partial x_i}\right)\left(\frac{\partial^2 h}{\partial x_j^2}\right) E\left\{[x_i - E(x_i)][x_j - E(x_j)]^2\right\}$$

$$+2 \sum_{\substack{i\;j\\ i\neq j}} \left(\frac{\partial h}{\partial x_i}\right)\left(\frac{\partial^2 h}{\partial x_i \partial x_j}\right) E\left\{[x_i - E(x_i)]^2\,[x_j - E(x_j)]\right\}$$

$$+2 \sum_{\substack{i\;i\;s\\ i\neq j\neq s}} \left(\frac{\partial h}{\partial x_i}\right)\left(\frac{\partial^2 h}{\partial x_i \partial x_s}\right) E\left\{[x_i - E(x_i)][x_j - E(x_j)][x_s - E(x_s)]\right\}$$

Higher order terms can also be calculated (reference 3, page 256 and reference 6).

If the equation variables are all statistically independent $(f(x_i|x_j, x_s) = f(x_i))$, then the Taylor series expansion simplifies to

$$E(z) \cong h\,[E(x_1), E(x_2), \ldots, E(x_n)] + 1/2 \sum_{i=1}^{n} \frac{\partial^2 h}{\partial x_i^2} \operatorname{Var}(x_i)$$

and

$$\operatorname{Var}(z) \cong \sum_{i=1}^{n} \left(\frac{\partial h}{\partial x_i}\right)^2 \operatorname{Var}(x_i) + \sum_{i=1}^{n} \left(\frac{\partial h}{\partial x_i}\right)\left(\frac{\partial^2 h}{\partial x_i^2}\right) \mu_3(x_i).$$

These expressions are approximations in that they ignore higher order terms by assuming they equal zero.

The partial derivatives, which are evaluated at the means, must exist for the method to be used. Similarly, the resulting expected values must be finite. These requirements can be satisfied for most physical systems.

2.3 SENSITIVITY ANALYSIS

When test validation of predictions is not feasible, and when the prediction technology is so complex that closed form expressions for prediction equations are not available, then prediction uncertainty must be estimated using other methods. An example of such a case might be an electrical network analysis using computer-aided techniques. The methods which can be used to determine prediction uncertainty include the joint or separate use of worst-case analysis, sensitivity analysis, or simulation techniques. The approach consists of repeatedly assuming values for the variables of the analysis system and evaluating the resultant prediction variability.

Let the system prediction for a nuclear effect, Z, and the analysis variables x_1, x_2, ..., and x_n, be related by the implicit function $Z = h(x_1, x_2, \ldots, x_n)$.

2.3.1 Worst-Case Analysis

A worst-case analysis approach may save time if the purpose of the analysis is to eliminate the necessity of considering a nuclear effect during further analyses. The approach is formulated to show that, even if the x_i values all take on unfavorable extreme values, the resulting worst-case system prediction (Z_w) allows for discontinuing further consideration of the effect.

The following simple procedure can be used for worst-case analysis under the conditions that: 1) the nature of the possible variations in x_i are known for all i, and 2) the direction of change in Z for a given change in x_i is known and interaction effects between variables are not strong enough to fully counteract this change.

Assume for expository simplicity that the worst case Z is the maximum Z that could reasonably result from a set of x_i, i=1,n. The worst case x_i is obtained using the following procedure: for each x_i, if $\frac{\Delta Z}{\Delta x_i} > 0$, pick $x_{i\gamma}$ such that the probability of $\{x_i > x_{i\gamma}\}$ is γ; if $\frac{\Delta Z}{\Delta x_i} < 0$, pick $x_{i\gamma}$ such that the probability of $\{x_i \leq x_{i\gamma}\}$ is γ where γ is arbitrarily selected and denotes the level of significance for the worst case analysis. Thus, if one must include all x_i - parameters which potentially affect Z, to a 0.99 confidence level, then γ is choosen as $1 - \gamma = 0.99$ or $\gamma = 0.01$. As noted above, the $x_{i\gamma}$ values cannot be determined unless the nature of the possible variations in x_i are known. As an example, if x_4 is distributed uniformly with a mean of $\bar{x}_4$ and a half-width of α then, for $\gamma = 0.05$, $[x_4 \leq x_{4\gamma}]$ and $[x_4 \geq x'_{4\gamma}]$ where $x_{4\gamma} = \bar{x}_4 + 0.9\alpha$ and $x'_{4\gamma} = \bar{x}_4 - 0.9\alpha$.

Calculate $Z_w = h(x_{1\gamma}, x_{2\gamma}, \ldots, X_{n\gamma})$ which is the worst-case estimate of Z. The probability of $\{Z > Z_w\}$ is not greater than γ; that is, Z is less than or equal to Z_w with probability greater than or equal to $(1 - \gamma)$. Since the confidence coefficient associated with a confidence interval is simply the

probability that the quanitity is included in the interval, then the confidence interval $\{Z \leq Z_W\}$ is associated with the confidence coefficient equal to, or greater than, $(1 - \gamma)$.

The confidence coefficient may be much greater than $(1 - \gamma)$ if several x_i have approximately the same $\frac{\Delta Z}{\Delta x_i}$, or may be quite close to $(1 - \gamma)$ if one x_i dominates changes in Z. The exact value for the confidence coefficient, or of the variance of the predictions, need not be known since it is only necessary to demonstrate that the upper bound of possible predictions is sufficiently small and that the nuclear effect can be ignored.

2.3.2 Sensitivity Analysis

Sensitivity analyses are used to identify any x_i which has a negligible effect on Z. If several x_i variables can be ignored, one of the other analyses techniques for determining data quality may become practical. As an example, simplification may allow use of closed form equations that permit calculation of system moments.

The following simple procedure can be used for sensitivity analysis under the conditions that: 1) the nature of the possible variations in x_i are known for all i being analyzed, and 2) the derivative $\frac{dz}{dx_i}$ is less than or equal to zero for all values of x_i, or is greater than or equal to zero for all values of x_i (i.e., monotonically increasing or decreasing).

To implement the analysis, select x_{iL} and x_{iu} values such that the probabilities of $\{x_i \leq x_{iL}\}$ and $\{x_i > x_{iu}\}$ are each equal to α. Following this selection process, calculate

$$\Delta Z_j = h(x_1, x_2, \ldots, x_{ju}, \ldots, x_n) - h(x_1, x_2, \ldots, x_{jL}, \ldots, x_n),$$

where the value for x_i, $i \neq j$, is numerically close to the expected value of x_i. The

median can be used for most distributions and, if the distribution is symmetric, the mean value, $(x_{iu} - x_{iL})/2$, can be used as the value of x_i. As an example, let $Z = 3x_1^2 + 2x_2x_3 - x_4$ where $x_1, x_2, \ldots x_4$ are symetrically distributed. Then

$$\Delta Z_1 = (3x_{1u}^2 + 2\bar{x}_2\bar{x}_3 - \bar{x}_4) - (3x_{1L}^2 + 2\bar{x}_2\bar{x}_3 - \bar{x}_4) = x_{1u}^2 - x_{1L}^2$$

and

$$\Delta Z_3 = (3\bar{x}_1^2 + 2\bar{x}_2x_{3u} - \bar{x}_4) - (3\bar{x}_1^2 + 2\bar{x}_2x_{3L} - \bar{x}_4) = 2\bar{x}_2 (x_{3u} - x_{3L})$$

To complete the analysis, determine the maximum ΔZ_i, denoted as ΔZ_{max}, for all i, i=1, ..., n.

The criteria for determining which variables can be ignored must be developed based on analysis requirements for the system. An example criteria is:

$$\text{if } \left| \frac{\Delta Z_i}{\Delta Z_{max}} \right| < \frac{1}{10\ n}$$

then delete variations in the parameter denoted by x_i from consideration.

The procedure as stated above ignores interaction effects. If a particularly strong interaction effect is suspected, it can be similarly evaluated as $\Delta Z_{ij} = h\ (x_1,x_2,\ldots,x_{iu},\ldots,x_{ju},\ldots,x_n) - h\ (x_1,x_2,\ldots,x_{iL},\ldots,x_{jL},\ldots,x_n)$, and both x_i and x_j should be retained if $(\Delta Z_{ij} - \Delta Z_i - \Delta Z_j)$ is not negligible.

2.3.3 Simulation

If the degree of uncertainty in the values of x_i is such that the values may be characterized as random variables selected from a population following a definable probability distribution, simulation techniques may be used to estimate the prediction uncertainty of Z. This technique is feasible for complex prediction technologies, especially those that are computer based.

The basic technique consists of using random number generators to create sample vectors $v_j = (\hat{x}_1, \hat{x}_2, ..., \hat{x}_n)$ for which $\hat{x}_i$ is a value generated from the distribution characterizing x_i. After determining the v_j vector, $Z_j = h(v_j)$ is calculated. This process is repeated m times; the distribution of the m generatred values of Z_j can then be used to characterize the uncertainties in the prediction technique. The technique can be used for cases where x_i and x_k are not independent if the random number generator is designed to generate correlated random numbers.

Uncertainties as to the true relational form can be handled using this technique. If the subjective estimate of the probability that h_1 is the true form for h is ϕ_1 and the probability that h_2 is the true form for h is ϕ_2; then m_1 and m_2 samples can be run using models h_1 and h_2, respectively, where $m_1/m_2 = \phi_1/\phi_2$. The resulting distribution of $(Z|m_1, m_2)$ would then include prediction variability due to uncertainty as to the true functional form.

Gathering data on distributions of input parameters, and performing each of the m iterations in a simulation study, can be quite expensive. Preliminary sensitivity analysis to eliminate variables may be beneficial in reducing the data analysis required, and may allow for the implementation of an approach other than simulation. Preliminary analysis to define the required precision, or acceptable bounds for the estimate of the variance, will help limit the required number of simulation iterations.

3.0 PROBABILITY OF SURVIVAL FOR INTEGRATED EFFECTS

3.1 DEFINED SCENARIO METHOD

One method of determining probability of survival is to hypothesize a scenario with one or more bursts at specified locations, and to analyze the probability of survival for these conditions. The scenario analysis described here is appropriate for a single node, link, or any other simple serial system.

More complex systems of redundant nodes and links, which have non-zero or non-one probabilities of survival, require further analysis by simulation or application of network or reliability theory (Appendix B).

3.1.1 Environment Characterization

The parameters given to characterize the burst must be sufficient to define each nuclear effort to be studied. Typical parameters would include latitude, longitude, height, fission yield, time of day, surface material composition, and possibly status of previous burst effects or the number of simultaneous bursts. The phenomenological sciences of physics or nuclear test history can then be applied to this burst characterization to deduce the magnitude of any of several effects E_j, $j=1, \ldots, e$.

The analysis is effectively a simulation of the attack scenario. In the real situation there is uncertainty both in targeting and design, and in realizable targeting accuracy and construction ability. The scenario method usually bypasses these uncertainties by treating the burst parameters as given values of the scenario.

Expressed in other terms, the analysis of the original problem,

$$P_r \{\text{Survive} \mid \text{Attack Scenario}\},$$

is made more tractable by substituting for it the scenario based analysis of

$$\{P_r \{\text{Survive} \mid \text{Attack scenario with given parameters}\},$$

but the variability in the estimate of the probability of survival is more properly stated with reference to the original problem. This statement of variability may be impossible to define objectively or qualitatively, but the results of the analysis should reflect at least qualitatively the implied uncertainty from using the substitute analysis. If it is desired to examine the implications of the uncertainty in the burst, several different scenarios may be hypothesized and separately analyzed.

3.1.2 Nuclear Effects

A burst has many interrelated effects which can affect a node of a communication system. Blast, shock waves, thermal ignition or melting, transient radiation, trapped radiation, electromagnetic pulse (EMP), system generated EMP (SGEMP) from gamma or X-rays, dust and debris, ejecta from ground bursts, magnetohydrodynamics (MHD), and particle fallout are effects that may separately or jointly affect nodes or links.

The effects may result in physical impairment of facilities and equipment, impairment of signal propagation or noise characteristics, or disabling of personnel. The system elements affected can include nodes such as land facilities, aircraft, and satellites; network links such as landlines; or the atmosphere, the ionosphere, or free space.

3.1.3 Integration of Assessment Results

An analysis of survivability of a node is facilitated by defining a matrix describing subunits of equipment versus the separately definable nuclear effects. For any one of these combinations, let Q(i,j) be the probability of survival of equipment i under nuclear effect j. Exactly what constitutes "survival" must be carefully defined for each unit of equipment in terms of operation of the node or link. In general, survivability is properly characterized as a probability, due to variability in parameters of the equipment and protective features, and due to analytical uncertainties in estimating thresholds and attenuation of nuclear effects within the node. For analysis of survivability, knowing the source of the uncertainty is not critical, though knowing the source may become important for decisions as to whether to harden the installation or perform a more precise analysis.

An over-simplified model for the probability of survival, Q(i,j), versus an independent variable that modulates the nuclear effects (range, for example) may be given as a step function as illustrated in Figure 3.1-1.

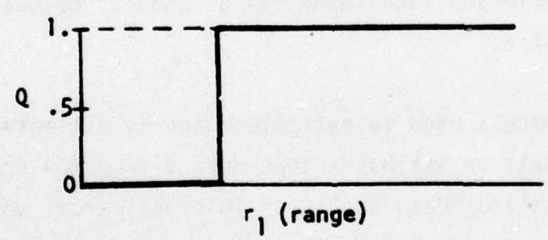

Figure 3.1-1. Step survivability function.

Taking into account variability in parameters of components between units and over time of day, temperature, humidity, age, etc., the model for Q can be represented as an S curve as shown in Figure 3.1-2. In this model there are random variations in the true range at which the equipment will survive, such that $r_{est} = R_{true} + \rho$, where ρ is a random variable which has a probability distribution that may be estimated by analysis, simulation, or test.

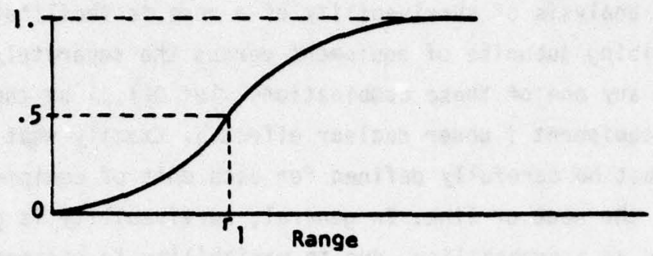

Figure 3.1-2. S-shaped survivability function.

If the available data on physical variability is defined in terms of physical units such as volts, or in terms of decibels of error about a mean safety margin, then to analyze variability of the range threshold requires a transformation of variables. Let r be the range threshold, where $r = h(x_1, x_2, \ldots, x_n)$ and the x_i are the physical variables or safety margins. Then, if the probability distributions of the x_i are known, the Method of Moments or Monte Carlo simulation techniques may be used to calculate the probability distribution of r.

The analytical models used to estimate Q and to estimate the requisite intermediate variables result in estimates that depart from the true state of nature by an uncertainty called "data quality." This analytical uncertainty may be due to modeling approximations, errors in stated input parameters, modeling oversights, etc. The model for Q may be conceptualized to include analytical uncertainty explicitly by modeling the R at a given value of Q as $R_{true} = r_{est} + \rho + \varepsilon$ where ρ is physical variability, as defined above, and ε is a random variable that represents the analysis uncertainty. This model can be used to put bounding curves on Q, as illustrated in Figure 3.1-3.

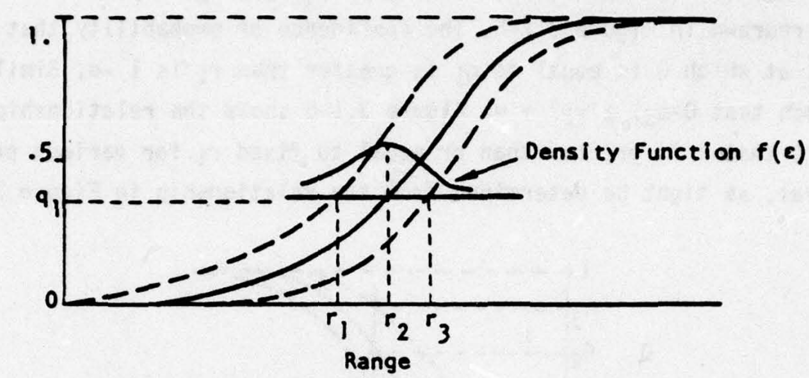

Figure 3.1-3. Bounded S-shaped survivability function.

An example interpretation of Figure 3.1-3 is that the probability of survival, Q, is estimated to be q_1 at a range of r_2, and in view of the analytical uncertainties, the true range at which Q is equal to q_1 is less than r_3 with confidence of $1 - \alpha$. In addition, there is a probability α that the range at which Q is equal to q_1 is less than r_1. Figure 3.1-4 shows the relationship between range and the confidence that R is less than or equal to r at a fixed probability of survival, q_1.

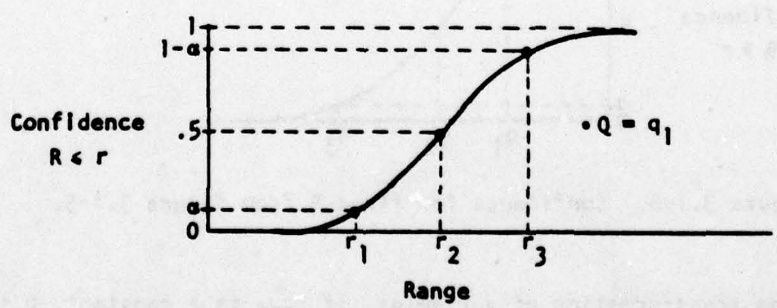

Figure 3.1-4. Confidence for fixed Q from figure 3.1-3.

Another interpretation for a given range, r_1, may be made from Figure 3.1-3 as redrawn in Figure 3.1-5. The confidence or probability that the true range, R, at which Q is equal to q_1 is greater than r_1 is $1 - \alpha$. Similarly, $Pr\{(R \text{ such that } Q=q_3) \geq r_1\} = \alpha$. Figure 3.1-6 shows the relationship for the confidence that R is greater than or equal to fixed r_1 for various probabilities of survival, as might be determined from the relationship in Figure 3.1-5.

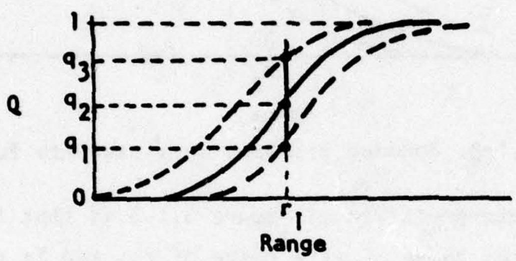

Figure 3.1-5. Confidence bounds; fixed R.

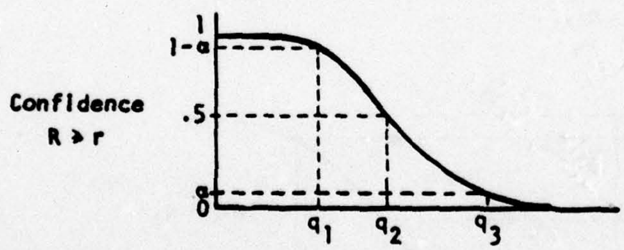

Figure 3.1-6. Confidence for fixed R from figure 3.1-5.

By the transformation of variables, if r_{est} is a constant, ρ is a random variable with known distribution, and ε is a random variable with known distribution from prediction data quality studies, then $R_{true} = r_{est} + \rho + \varepsilon$ is a random variable with a definable statistical distribution function. In

particular, if ρ is distributed as normal with mean 0 and standard deviation σ_ρ (abbreviated as ρ d.a. $N(0,\sigma_\rho)$), and ε d.a. $N(0,\sigma_\varepsilon)$, then R d.a. $N(r_{est}, (\sigma_\rho^2 + \sigma_\varepsilon^2)^{1/2})$ which defines an overall probability of survival curve $Q(i,j)$ that incorporates both physical variation and analytic uncertainty.

Let $Q(.,.)$ be the probability of survival of the entire node under all effects, with values of the independent variables that modulate the nuclear effect given by the scenario. Under the simplifying assumptions that survival of each subunit of equipment is statistically independent from the survival of any other subunits, and survival under the stress of each effect is statistically independent of the survival under the stress of other effects, and all subunits are necessary serial parts of the system that constitutes the node, the effects may be integrated using the relationship

$$Q(.,.) = \prod_{j=1}^{e} \prod_{i=1}^{s} Q(i, j). \qquad (3.1\text{-}1)$$

This probability of survival formulation is useful for first order approximations even if independence is a questionable assumption. As an example, if any $Q(i, j)$ is 0, the node will not survive. Prior to making completely detailed calculations, if one subunit of equipment is suspected to be vulnerable to a nuclear effect, analysis of the subunit may suffice to demonstrate vulnerability of the entire node.

Similarly, to investigate the survivability of one subunit of equipment using the relationship ($Q(i,.) = \prod_{j=1}^{e} Q(i, j)$), it may suffice to investigate only the nuclear effect j which is suspected of being the most damaging or for which results are readily calculated and suspected to give a near zero probability of survival. For example, a long analysis of ground shock effect on an antenna is not important for a survivability study if a two minute calculation shows a high probability that the antenna is damaged by heat.

The assumption of independence between probabilities of survival may be valid for many effects and subunits of equipment. In other cases, strong dependence may be expected. If, for example, the subunits are electronic units in the same building and effect j is blast dynamic pressure, and if the building collapses and crushes box i, there is very high probability that box i + k will also be crushed. For this example, $Q(i + k, j\ i, j) \simeq 1$; that is, the probability of survival of equipment unit i + k to effect j, given the survival of equipment unit i to effect j, is close to one.

As another example, analytical and physical uncertainty errors for blast may be expected to be positively correlated with similar errors developed in analyzing for shock effects, due to a common relation to physical strength. Here $Q(i, j + h \mid i, j)$ is expected to be greater than the unconditioned $Q(i, j + h)$.

The analysis of the node probability of survival under assumed statistical dependence can become complicated if there are significant numbers of values of Q which are neither nearly 0 nor nearly 1. The sequence of conditional events and their careful definition become important for these situations. For an assumed serial set of equipment,

$$\begin{aligned} Q(.,.) = {} & Q(1,1)*Q(1,2 \mid \{1,1\})*Q(1,3 \mid \{1,1\},\{1,2\})*Q(1,4 \mid \{1,1\},\{1,2\},\{1,3\})* \ldots \\ & *Q(2,1 \mid \{1,.\})*Q(2,2 \mid \{1,.\},\{2,1\})*Q(2,3 \mid \{1,.\},\{2,1\text{-}2\})* \ldots \\ & *Q(\{i,j\} \mid \text{all } \{k,h\} \text{ where } [k < i] \text{ or } [k = i \text{ and } h < j])* \ldots. \\ & *Q(\{s,e\} \mid \text{all } \{k,h\} \text{ where } k \le s,\ h < e) \qquad (3.1\text{-}2) \end{aligned}$$

where * denotes multiplication and the event i,j is the event that subunit i survives the isolated subeffect j.

3.1.4 Example

To illustrate the above scenario method of integrating nuclear effects for probability of survival estimation, the following example with hypothetical values and relationships will be developed.

Let the scenario burst be a single 1 megaton burst at a height of 100 meters with a .X fission fraction and a Y.Y KEV X-ray temperature and a .ZZ X-ray fraction. Assume the burst is over level land at a latitude of 48° 21'N and a longitude of 122° 40'W at 1433 hours GMT on 2/31/84. The surface is grass covered land overlaying glacial deposits of moderate sized gravel and clay, over a point at a horizontal range of 150 meters due north from the center of a target facility.

Let the node being assessed consist of all equipment necessary to perform high frequency (HF) receive and transmit functions, operating out of a rectangular, above-ground concrete slab building with reinforcing bar, dimensioned 30 meters, north to south, 40 meters, west to east, and 10 meters high. Let the burst be part of a surprise attack with no special preparations or readiness.

Let the subunits of equipment be 1) the antenna, 2) the multiplexer, 3) the receiver, 4) the teletype, 5) the transmitter, and 6) the power supply. Let the equipment be operating at H hertz, using FSK over 50 channels, all equipment on line without ready spares.

Let survival of the node at any point in time be defined as the ability to transmit and receive 10 or more channels with 90% of full signal power and bit error rate < 1/100 bits. Assume that to meet this definition of survival all subunits of equipment must be operating.

Let the total effect be analyzed in terms of the subeffects blast, shock, thermal, radiation, dust, ejecta, EMP, MHD, and psychological and physical effects to personnel. The matrix of equipment and effects is as defined in Table 3.1-1.

Table 3.1-1. Basic equipment/effect analysis table.

Equipment	1 Blast	2 Shock	3 Thermal	4 Radiation	5 Dust	6 Ejecta	7 EMP	8 MHD	9 Personnel
1) Antenna									
2) Mux									
3) Rcvr									
4) TTY									
5) XMTR									
6) PS									
7) Personnel									

As an example of the analysis leading to the characterization of one matrix entry, a hypothetical analysis of the effect of neutron and gamma radiation on the transmitter will be presented. Let the fluence be b units/cm^2 at the building. Analysis predicts the attenuation to be 30 dB if the transmitter is in electronics rack 4. The prediction accuracy is characterized by prediction error that is normally distributed with mean 0 dB and standard deviation 10 dB. The transmitter response to radiation is permanent degradation of a semiconductor component which has the functional effect of complete transmitter failure if neutron fluence of the component exceeds approximately S units/square cm. The logarithm of this failure level varies among similar semiconductors and transmitters by an approximately normally distributed variate N(0,4).

It is convenient to characterize the radiation effect in terms of a safety margin (SM) where SM = 20 log (threshold/threat magnitude). The estimate of the mean value of safety margin is SM = 20 log (S) - (20 log b - 30) where the true unknown safety margin is normally distributed about this estimate with standard deviation

$$\sqrt{\sigma_s^2 + \sigma_{b1}^2} = \sqrt{4^2 + 10^2} = 10.8 \text{ dB.}$$

This follows from the fact that if X d.a.N(μ_X,σ_X) and Y d.a.N(μ_Y,σ_Y), then X ± Y d.a.N($\mu_X \pm \mu_Y,(\sigma_X^2 + \sigma_Y^2)^{1/2}$). Therefore, if SM = 8 dB, then Pr{SM > 0} = Q(5,4) = .77 for the transmitter with radiation considered alone as illustrated in Table 3.1-2.

Assuming the above type of information is available and the analysis is performed for all the effects and equipment, the matrix can be filled in with all the unconditional probabilities Q(i, j) as illustrated in Table 3.1-2.

Table 3.1-2. Example equipment effect analysis.

	Effect									
	1	2	3	4	5	6	7	8	9	
Equipment	Blast	Shock	Thermal	Radiation	Dust	Ejecta	EMP	MHD	Psych	Q(i,.)
1) Ant	.95	.7	.8	1	1	.99	.99	1	1	.52
2) Mux	.8	.9	.48	.5	1	.99	.4	1	1	.06
3) Rcvr	.8	.9	.97	.98	1	.99	1	1	1	.68
4) TTY	.8	.9	.99	1	1	.99	1	1	1	.71
5) XMTR	.8	.9	.8	.77	1	.99	1	1	1	.44
6) PS	.8	.9	.99	1	1	.99	1	1	1	.71
7) Personnel	.75	.9	.9	.9	1	.99	1	1	.83	.45
Q(.,j)	.23	.37	.26	.34	1	.93	.4	1	.83	.002=Q(.,.)

The entries in Table 3.1-2 allow for the definition of a first-order approximation to the probability of survival of the node assuming independence. For this example, as defined in equation 3.1-1, Q(.,.) = .002. If Q(.,.) had been 1, or if all vulnerability is due to one term or to statistically independent terms, the first-order approximation analysis would be sufficient.

Assuming that there are some known dependencies between effects and between equipments, a matrix of conditional probabilities of survival,

$$Q(\{i,j\} \mid \text{all } \{k,h,\} \text{ where } [k < i] \text{ or } [k = i \text{ and } h < j]),$$

can be created as illustrated in Table 3.1-3.

Table 3.1-3. Statistical dependence example of equipment effect analysis.

	Effect									
	1	2	3	4	5	6	7	8	9	
Equipment	Blast	Shock	Thermal	Radiation	Dust	Ejecta	EMP	MHD	Psych	Q'(i,.)
1) Ant	.95	1	.8	.1	1	.99	.99	1	1	.745
2) Mux	.8	1	.48	.9	1	.99	.4	1	1	.137
3) Rcvr	.99	1	.99	.99	1	.99	1	1	1	.961
4) TTY	.99	1	.99	1	1	.99	1	1	1	.970
5) XMTR	.99	1	.99	.95	1	.99	1	1	1	.922
6) PS	.99	1	.99	1	1	.99	1	1	1	.970
7) Personnel	.95	1	.9	.99	1	.99	1	1	.95	.796
Q(.,j)	.694	1	.332	.838	1	.932	.396	1	.95	0.068

As an example of the interpretation of entries in Table 3.1-3, the {3,3} entry, .99, is the probability that the receiver survives thermal effects given that it survives shock and blast effects, and given that the antenna and multiplexer survive all effects. From entries in this table, an estimate of the probability of survival of the node can be calculated using Equation 3.1-2; the result is $Q'(.,.) = .068$ where $Q'(.,.)$ is the probability of survival of the node containing serially dependent equipment for which the probabilities of survival are statistically dependent.

The calculation of the conditional probabilities may be straightforward based upon logical and physical dependencies such as: "If the TTY survives blast, then the TTY will survive shock." Other calculations of conditional probabilities may require substantial modeling investigation of covariances among variables, from information such as: "The dB errors in analysis of attenuation of thermal effects have a correlation coefficient $\rho = .8$ with dB errors in the analysis of attenuation of beta radiation." Such correlation data might be used to calculate conditional probabilities, $Q(\{i,j\} \mid \{i,h\})$.

The value of $Q(\{i,j\} \mid \{i,h\})$ may be determined by Monte Carlo simulation. Correlated pseudo-random numbers may be generated for modeling uncertainty errors for j and h. The resulting $Q(\{i,j\})$ and $Q(\{i,h\})$ pairs that are so generated may in turn be used for generating pairs of Bernoulli random variables that may be used in a counting algorithm to estimate $Q(\{i,j\}$ and $\{i,h\})$. The average of these estimates of $Q(\{i,j\}$ and $\{i,h\})$ can be used to calculate $Q(\{i,j\} \mid \{i,h\})$ from the definition of conditional probability

$$Q\{i,j\} \mid \{i,h\}) = \frac{Q(\{i,j\} \text{ and } \{i,h\})}{Q(\{i,h\})}$$

It may be possible to determine the values of $Q(\{i,j\} \mid \{i,h\})$ using more direct mathematical calculations, a technique such as the method of moments for correlated variables, or the transformation of variables.

3.2 FUNCTION OF INDEPENDENT VARIABLE APPROACH

The various nuclear effects are individually and functionally dependent on several scenario-type independent variables such as range, altitude, etc. If one (or several) of the independent variables is selected for analysis, and the probability of survival is determined over the values of this variable with all other variables held constant, a useful conceptualization of vulnerability relationships may be obtained.

The following discussion will concentrate on the one variable approach illustrated in Figure 3.2-1. Extension to the n-variables would follow similar lines; such extensions are conceptually described in Figures 3.2-2 and 3.2-3.

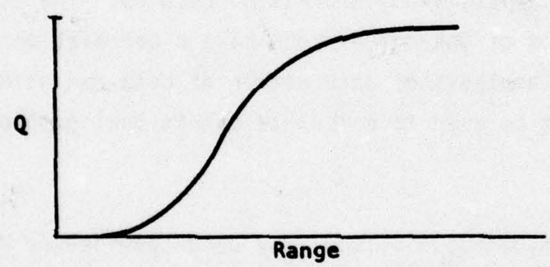

Figure 3.2-1. Example of one variable survival relation.

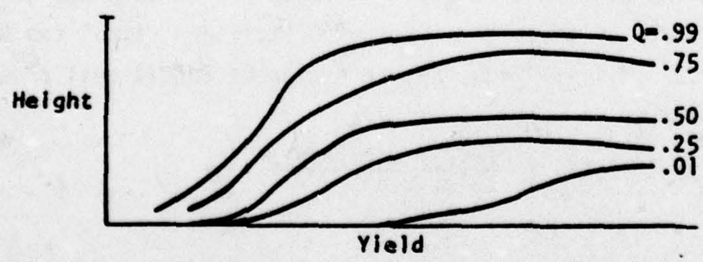

Figure 3.2-2. Example of two variable survival relation.

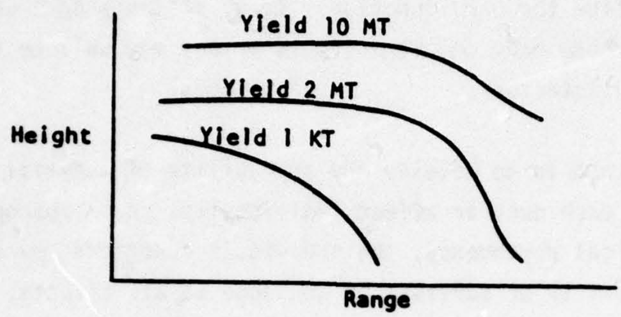

Figure 3.2-3. Example of three variable survival relation, Q=.5.

3.2.1 Integration of Assessment Results

The remarks presented in Section 3.1 regarding series structures and statistical dependence, and their impact on the form of survivability functions, apply also to the material presented in this section.

The "function of a variable" approach discussed here may use any of the independent variables. For concreteness of discussion, the particular independent variable assumed here will be the range from target to actual ground zero. This parameter was chosen for the discussion due to its dominant role in determining the magnitude of several of the effects from the INCA list of nuclear effects, which includes blast, thermal, radiation, EMP, shock, dust, ejecta, and MHD. Having analyzed the probability of survival as a function of range allows ready evaluation under various assumptions of targeting accuracy, and the analysis can be helpful in evaluating probable collateral effects on the site if it should happen not to be the designated target but is close to another target.

There is more than one method of creating the integrated probability of survival for multiple effects over the range of an independent variable. One method is to use the scenario method described in Section 3.1 and iterate the

scenario through the appropriate values of range to determine a set of relationships that define the Q(r) functional form. If the single scenario analysis logic has already been developed, this method may well be the most straightforward and satisfactory.

A second method is to develop the probability of survival as a function of range for each nuclear effect individually. Under appropriate conditions of statistical dependency, the individual functions may be combined to estimate the probability of survival of the node to all effects; this method is developed further in the following paragraphs.

For any one effect and unit of equipment, the threshold and threshold uncertainties must be determined as shown in Figure 3.2-4. The nuclear effect response and response uncertainties at the equipment, Figure 3.2-5, must also be determined. The probability of survival relationship, Figure 3.2-6, is determined as before from the threshold and response relationships together with the associated uncertainties.

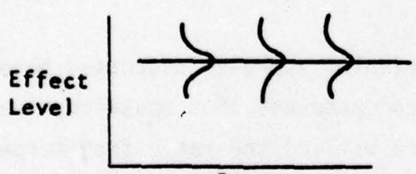

Figure 3.2-4. Threshold; $T_1 = h_1(r)$.

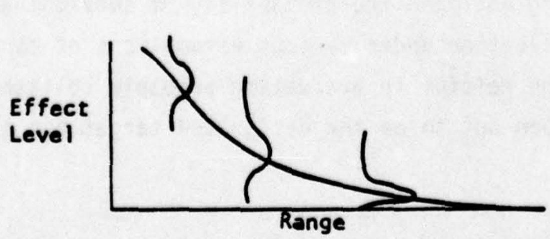

Figure 3.2-5. Effect response; $E_1 = g_1(r)$.

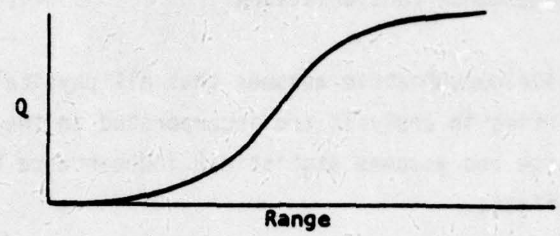

Figure 3.2-6. Probability of survival; $Q_1 = K_1\ (g_1(r), h_1(r))$, equipment 1.

Combining equipment 1 and equipment 2 under assumed statistical independence and series functional dependence gives an example of a system probability of survival for one effect as illustrated in Figure 3.2-7. Other effects may be analyzed in the same way, resulting in a set of Q.j for all effects j.

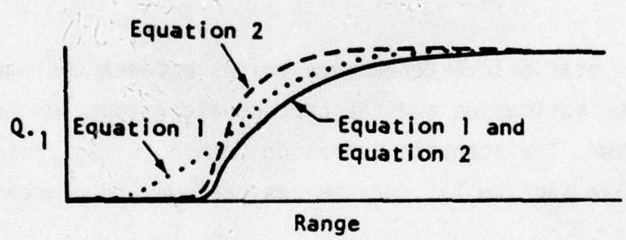

Figure 3.2-7. System of equipments; $Q._1 = \pi Q_{i1}$, effect 1.

To create an integrated probability of survival, under an assumption of statistical independence, effects may be combined as $Q.. = \Pi Q.j$, as diagrammed in Figure 3.2-8.

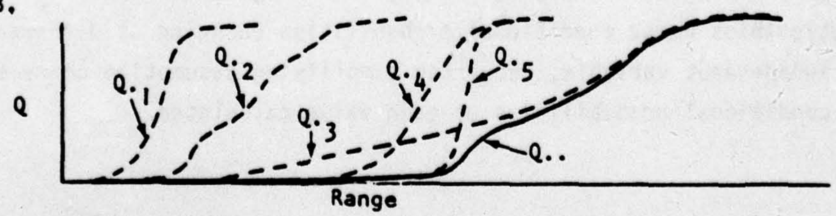

Figure 3.2-8. Integrated survival probability.

3.2.2 Statistical Dependence Considerations

The outline development above assumes that all physical variabilities and estimation uncertainties in analysis are incorporated in the survival probability function curve and assumes statistical independence between equipment and between effects.

If statistical dependencies exist between equipment or effects, the analytical inclusion of these dependencies will most likely be complicated. For the simple case where strong positive correlation exists between the probabilities of survival for n subcategories, n-1 of the values (curves) of Q can be omitted and a good approximation obtained. Probabilistically, this follows from the fact that if $P(B|A) \approx 1$ and $P(C|A \wedge B) \approx 1$, then $P(A \wedge B \wedge C) = P(A)P(B|A)P(C|AB) \approx P(A)$.

If moderate statistical dependence exists between equipment or effects, the parameter estimation and the correct algorithms for integration become more complicated. The scenario iteration method, using the dependence techniques described in Section 3.1, may be the best way to proceed.

For the independent variable approach, the correct dependency relationships must be analyzed from the covariation in the uncertainties in the predicted threat environments and the covariation in the uncertainties in the predicted thresholds, as described in Section 3.1. The mappings and transformations of variables into the conditional survival probabilities, $Q(\{i,j\}) \mid \text{all } \{k,h\} \ni k \leq i,\ h < j)$, will in general result in the relationships among conditional probabilities changing at different values of the independent variable, requiring simplifying assumption or re-evaluation of the conditional probabilities at each value calculated.

4.0 CONCLUSIONS

The determination of an overall assessment statement for a communication facility requires two steps. The first is the determination of the data quality and survival confidence for each specific nuclear effect, and the second is the integration of the information for various specific effects into the overall assessment statement.

Four techniques which can be applied to the determination of the data quality and confidence for a specific nuclear effect have been defined; they are:

1) A test verification technique which uses analytical predictions and test measurements.

2) A method of moments technique which can be used whenever analytical expressions are available.

3) A sensitivity analysis technique which can be used to exclude from further consideration components of a facility which are unaffected by the "worst case" of a specific nuclear effect; the technique can also be used to select the combinations of components and nuclear events which are most significant with respect to survival confidence.

4) A full scale or limited simulation technique which, although attractive because it is an analytical simulation of a complete test, suffers from errors in an imperfect model, and is often the most expensive way of obtaining the required data quality/survival confidence information.

Two techniques have been defined which can be applied to the integration of the information for various specific effects into an overall assessment statement; they are:

1) A defined scenario technique which specifies the environment characteristics such that particular nuclear effects are defined in a sequence so that the integrated assessment results can be presented in the form of a time history.

2) A function of independent variable technique which provides for investigations with all but one of the variables held constant. This technique provides insight into the conceptual nature of the functional dependencies that are indepedent of the scenario.

All the techniques defined within this report have assumed statistical independence; in some cases the effect of statistical dependencies are illustrated. When the assumption of statistical independence is invalid, standard techniques for calculating covariance and correlation must be employed. When these determinations are computationally impractical, scenario iteration can be used.

5.0 RECOMMENDATIONS

1) The existing INCA data base should be utilized, and expanded as required, to allow for a comparison and evaluation of the various techniques defined in this document.

2) The statistical characterization of the uncertainties (i.e. data quality) should be determined and used whenever predictions are developed. The theoretical foundations of the data quality concept were documented in "Survivability/Vulnerability Safety Margin Assessment," DNA 3859Z, September 1975, and were expanded in "Response Prediction Characteristics, Data Quality and Related Topics," D194-10021-1, September 1977. These results, as well as those results and techniques presented in other documents, reports and papers addressing the subject, should be documented in a self-contained discussion in a form readily comprehensible to nonstatisticians and to readers entirely unfamiliar with the data quality concepts.

3) The material contained in the documents discussed in 2) above should be used to prepare lecture notes and visual aids to be used during seminars in assessment techniques. This material would be designed for nonspecialists and would include the elementary statistics required to develop and apply the data quality concept. In addition, the material would serve to establish a unified language and notation for the several contractors and governmental agencies associated with the INCA Program.

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APPENDIX A

REQUIRED SAMPLE SIZE FOR VARIANCE ESTIMATION

Given a random variable, X, it is then desired to characterize the spread in the distribution of X. A sample of n values of x_i is to be taken to provide data; thus, the required value of n must be determined. Though range or interquantile range may be used to characterize spread, the most commonly used statistic for spread is the variance, or second moment about the mean.

The population variance of X, σ_X^2, is defined by $\sigma_X^2 = \int_{-\infty}^{\infty} (X-\mu_X)^2 f(X)\,dx$ for continuous distributions or $\Sigma_X (X - \mu_X)^2 f(X)$ for discrete distributions. The sample information may be used to estimate the population variance. An unbiased estimate of variance is $\hat{\sigma}^2_X = s^2 = \frac{\Sigma(X_i - \overline{X})^2}{n - 1}$.

To determine what sample size is needed to estimate the variance with a given precision, the relationship among σ_X, s, and n must be known. If the form of a population distribution is known, or can be hypothesized and shown to be a reasonable assumption or approximation, the relationship between σ_X, s, and n may be calculated. In particular, if X is a normally distributed random variable with mean μ_X and standard deviation σ_X, and a sample of size n from the population of X gives a variance estimate s_X^2, then the random variable $(n-1)\, s_X^2$ has a χ^2 distribution with n-1 degrees of freedom. Therefore, if it is desired to select a sample size large enough so there is only a probability α that S_X is less than σ_X by more than p%, or $Pr \left\{ \frac{\alpha_X - S_X}{\alpha_X} \geq \frac{p}{100} \right\} = \alpha$, then $Pr\, \{\chi^2_{n-1} \leq (1 - p/100)^2 (n-1)\} = \alpha$. The solution then consists of finding the smallest *n* such that $\chi^2_{n-1},\ \alpha \leq (1-p/100)^2(n-1)$.

For example, to assure a 0.05 probability or less that S_X is less than σ_X by more than 10%, a sample size close to 200 is needed. To assure a 0.10 probability or less that S_X is less than σ_X by more than 25%, a sample size of 16 is required.

The normal distribution case may be used for an approximation of required sample sizes for variance estimation for other distributions, if they are approximately normally distributed. Exact probability distributions of the sample variance for non-normal distributions are not widely published. Alternatively, distribution-free methods may be used to place confidence bounds on population spreads based upon sample data, though due to their generality they can result in unrealistically large bounds or large sample sizes.

APPENDIX B
SYSTEM SURVIVAL

B.1 SERIES SYSTEM SURVIVAL

If a system is a functional series of units, every unit in the system must function in order for the system to function, and the unit survival probabilities are statistically independent, then the probability of system survival is the product of the unit probabilities of survival; that is,

$$Q_s = Q_1 \cdot Q_2 \cdot Q_3 \cdots Q_n = \prod_{i=1}^{n} Q_i$$

where

Q_s = probability of system survival
Q_i = probability of unit "i" survival

B.2 PARALLEL SYSTEM SURVIVAL

If a system is a functional combination of units in parallel, so that the system will function if any unit functions, and the unit survival probabilities are statistically independent, then the probability of system survival is a function of a union of events,

$$Q_s = P_r \left\{ \begin{array}{c} \text{unit 1 survives OR unit 2 survives OR } \ldots \\ \ldots \text{ OR unit n survives} \end{array} \right\}$$

$$= 1 - \prod_{i=1}^{n} (1 - Q_i)$$

B.3 K OUT OF N SYSTEM

If a system operates whenever at least k out of n units operate, and the unit survival probabilities are statistically independent, then the probability of system survival is the sum of the probabilities of survival for combinations k or more surviving units; that is,

$$Q_s = \sum_{i=k}^{n} \sum_{\substack{\text{all combinations} \\ \text{of } i \text{ survivors}}} \left[\prod_{j \varepsilon i} Q_j \prod_{j \not\varepsilon i} (1 - Q_j) \right]$$

where "ε" denotes that j is an element of i and "$\not\varepsilon$" denotes that j is not an element of i.

In a simple case where n = 3 and k = 2, the above equation simplifies to:

$$Q_s = Q_1Q_2 (1-Q_3) + Q_1 (1-Q_2) Q_3 + (1-Q_1) Q_2Q_3 + Q_1Q_2Q_3.$$

B.4 COMPLEX SERIES - PARALLEL SYSTEMS

Techniques for evaluating complex systems of parallel and series units can be found in design reliability textbooks (reference 8). Simple system combinations can be analyzed in a straightforward manner. For complex systems exact computation can be a formidable task; bounding algorithms or computer solution from event tree diagrams can be used in those cases.

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